

FLUIDS

FLUIDS AT REST

What you already know:

- **Fluids**—matter that flows such as gases and liquids
- **Phases or States of Matter**
 - **Solids**—molecules are *very close* together [due to strong *intermolecular* forces] with no translational or rotational motion, only vibration about fixed points. This means they are not very compressible and have a fixed shape, size, and volume.
 - **Liquids**—molecules have all three types of motion and are *close* together [still due to strong *intermolecular* forces], but can slide past one another [which is why they flow and have no fixed shape!]. They are still not very compressible and have a fixed volume but take on the shape of the container, yet may not fill the container.
 - **Gases**—molecules have all three types of motion, are very energetic and have overcome all *intermolecular* forces and are 2,000 molecular diameters farther apart than in either the solid or liquid phase where they are usually less than one molecular diameter apart. Gases expand to fill their container since there are no intermolecular forces acting to keep them attracted to one another. They are *very compressible*, have no fixed size, shape, or volume
 - **Plasma**—exist only at very high temperatures and consists of ionized atoms [not something we will focus on!]
 - **Colloids**—a suspension of solid particles in a liquid—some argue that this should be a separate state of matter, others consider it a liquid. If the particles in a mixture with a liquid solvent are too small to refract light, we call it a solution. If the particles are too large, you can't keep them in solution [so they don't hang around long enough to refract light] and they settle, we call that a suspension. A colloid is in between with regard to particle size and *does* refract light [Tyndall effect]. Milk is a colloid of fat particles suspended in a water and lactose solution. If you add chocolate syrup and stir you can make a suspension—the chocolate particles settle but the fat particles remain in solution!

DENSITY & SPECIFIC GRAVITY—you may already know this too!

- **Density**—mass per unit volume. We've used d as it's symbol in all your other science classes, physicists use the Greek letter "rho", ρ , since we use d for distance all the time!

$$\text{DENSITY: } \rho = \frac{m}{v} = \frac{\text{mass}}{\text{volume}}$$

- It follows that $\text{mass} = \rho v$ and $\text{weight} = mg = \rho v g$
- The SI unit for density is kg/m^3 [often reported in g/cm^3 and never forget that $1 \text{ cm}^3 = 1 \text{ ml}$]
$$\frac{1 \text{ kg}}{\text{m}^3} = \frac{1000 \text{ g}}{(100)^3 \text{ cm}^3} = \frac{1000 \text{ g}}{1,000,000 \text{ cm}^3} = 10^{-3} \text{ g / cm}^3$$
 - ^ a density in g/cm^3 should be multiplied by 1,000 to give the result in kg/m^3
- Temperature does have a slight effect on densities

TABLE 10–1
Densities of Substances[†]

Substance	Density, ρ (kg/m ³)
<i>Solids</i>	
Aluminum	2.70×10^3
Iron and steel	7.8×10^3
Copper	8.9×10^3
Lead	11.3×10^3
Gold	19.3×10^3
Concrete	2.3×10^3
Granite	2.7×10^3
Wood (typical)	$0.3\text{--}0.9 \times 10^3$
Glass, common	$2.4\text{--}2.8 \times 10^3$
Ice	0.917×10^3
Bone	$1.7\text{--}2.0 \times 10^3$
<i>Liquids</i>	
Water (4° C)	1.00×10^3
Blood, plasma	1.03×10^3
Blood, whole	1.05×10^3
Sea water	1.025×10^3
Mercury	13.6×10^3
Alcohol, ethyl	0.79×10^3
Gasoline	0.68×10^3
<i>Gases</i>	
Air	1.29
Helium	0.179
Carbon dioxide	1.98
Water (steam) (100° C)	0.598

[†]Densities are given at 0°C and 1 atm pressure unless otherwise specified.

Example 1

What is the mass of a solid iron wrecking ball of radius 18 cm?

- **Specific Gravity**—the ratio of $D:D$ water @ 4° Celsius [why 4°C?]
 - $D_{\text{water}} = 1.00 \text{ g/cm}^3 = 1,000 \text{ kg/m}^3$
 - That means that the specific gravity, SG, is the density without units [why?] **IF** the density is reported or first converted to g/cm

PRESSURE IN FLUIDS

- **Pressure**—Force per unit Area where the force is acting Z to the surface area

$$\text{Pressure: } P = \frac{F}{A}$$

- The SI unit is the Pascal [in honor of Blaise Pascal...more to come about him]
 - $1 \text{ Pa} = 1 \text{ N/m}^2 = 1.45 \times 10^{-4} \text{ lb/in}^2$ (psi)
 - A 60 kg person whose two “feetsies” cover an area of 500 cm^2 will exert a P of

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(60\text{kg})\left(\frac{9.8\text{m}}{\text{s}^2}\right)}{500\text{cm}^2\left(\frac{1\text{m}^2}{(100)^2(\text{cm})^2}\right)} = 12,000 \frac{\text{N}}{\text{m}^2}$$

- Same person stands on one foot and the area is cut in half which doubles the pressure, so
 $\hat{P} = 24,000 \text{ N/m}^2$

Fluids exert pressure in all directions

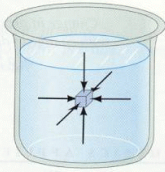
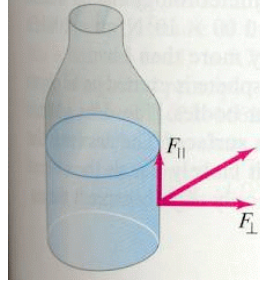


FIGURE 10-1 Pressure is the same in every direction in a fluid at a given depth; if it weren't, the fluid would be in motion.

FIGURE 10-2 If there were a component of force parallel to the solid surface, the liquid would move in response to it; for a liquid at rest, $F_{\parallel} = 0$.

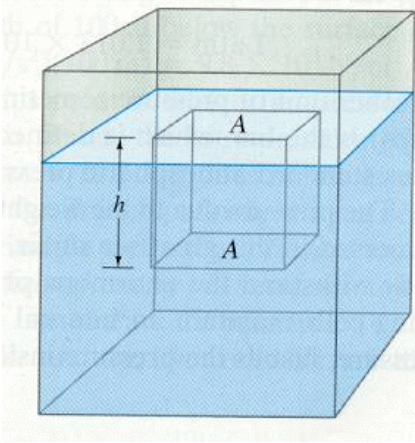


- **A fluid exerts a pressure in all directions**
- This is a well known fact to scuba divers!
- If a fluid is NOT flowing, it is at rest, then the P on all sides must be equal to keep it static.
- The force due to the fluid's pressure is \perp to any surface it is in contact with.
 - If there were a component of the force that is \parallel to the surface, then according to Newton's 3rd Law, the surface would exert a force *back* on the fluid that would also have a component \parallel to the surface.
 - If that were the case, it would cause the fluid to flow and it couldn't be at rest!

- $\hat{=}$ the F due to the P is \perp to the surface

- How does the pressure of a liquid of uniform density vary with depth?
 - consider a point at a depth , h, below the surface of a liquid [surface is at height, h, above this point]
 - P due to *fluid* at h is due to the weight of the column of *fluid* above it $\hat{=}$

FIGURE 10-3 Calculating the pressure at a depth h in a liquid.



$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

The A's cancel and you're left with:

$$P = \rho gh$$

- $\hat{=}$ P \propto ρ depth
- $\hat{=}$ P at EQUAL depths are the SAME!
- **incompressible**—density is constant and does NOT change with depth. Obviously the oceans are exceptions, this applies to MUCH smaller fluid samples! [gases are VERY COMPRESSIBLE and ρ varies with depth]
- IF ρ varies only slightly, then $P = \rho gh$

Example 2

The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the water pressure at the faucet.

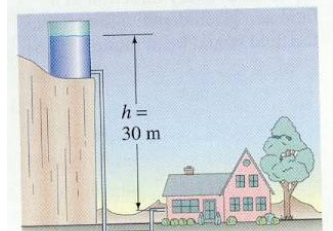


FIGURE 10-4 Example 10-2.

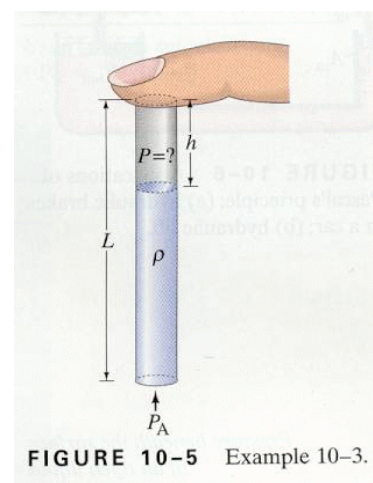
ATMOSPHERIC PRESSURE & GAUGE PRESSURE

The pressure of the Earth's atmosphere, as in any fluid, changes with depth

- P varies inversely with altitude. As altitude \uparrow , P \downarrow
- No distinct top surface to the Earth's atmosphere for measuring h
- Calculate approximate differences in P using $\Delta P = \rho g \Delta h$
- The weather also plays a factor!
- At sea level $1.013 \times 10^5 \text{ N/m}^2 = 1 \text{ atm} = 101.3 \text{ kPa} = 760 \text{ mm Hg} = 760 \text{ torr}$
- $1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2 = 100 \text{ kPa}$
- **GAUGE pressure**—P measured over and above atmospheric pressure—sort of like using the tare button on a balance to remove the mass of a weighing container
 - Absolute $P = P_{\text{atm}} + P_{\text{gauge}}$

Example 3

You insert a straw of length L into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw retains the liquid such that the distance from the bottom of your finger to the top of the liquid is h. Does the air in the space between your finger and the top of the liquid have a pressure P that is a) greater than, b) equal to, or c) less than the atmospheric pressure P_A outside the straw? Explain.



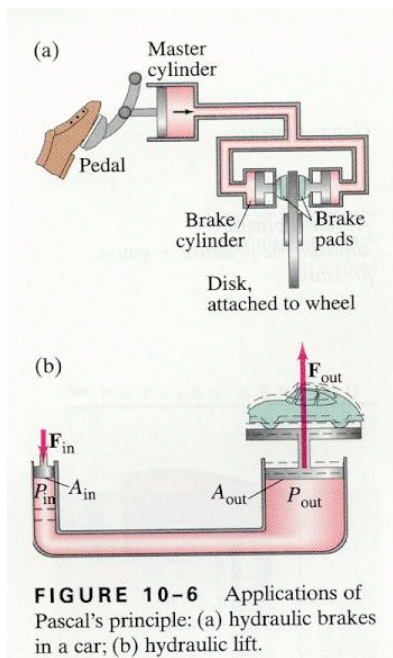
PASCAL'S PRINCIPLE

Blaise Pascal (1623-1662) stated that: ***Pressure applied to a confined fluid increases the pressure throughout by the same amount.***

- Earth's atmosphere exerts a pressure on ALL objects with which it is in contact, including other fluids & external P is transmitted through that fluid
 - The pressure due to water @ a depth of 100m below the surface of a lake is

$$\Delta P = \rho g h = (1,000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m}) = 9.8 \times 10^5 \text{ N/m}^2 = 9.7 \text{ atm for just H}_2\text{O!}$$
 Add the atmospheric pressure of the Earth's atmosphere and the pressure is 10.7 atm (at sea level)
- **hydraulics**—a small force can be used to exert a large F by making the area of one piston [the output] *larger* than the area of the other piston [the input]

- Assume the input & output pistons are at the same h so Pascal's principle applies



$$P_{out} = P_{in} \text{ thus}$$

$$\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}} \text{ AND}$$

$$\frac{F_{out}}{F_{in}} = \frac{A_{out}}{A_{in}}$$

- The quantity F_{out}/F_{in} is called the **mechanical advantage** and is equal to the ratio of the areas
- IF the area of the output piston is 20 x that of the input cylinder, the F is multiplied by 20; thus a force of 200 lbs could lift a 4,000 lb car! Think about this the next time you sit in the stylist's chair getting your hair cut and they raise and lower you!

MEASUREMENT OF PRESSURE; GAUGES & THE

BAROMETER

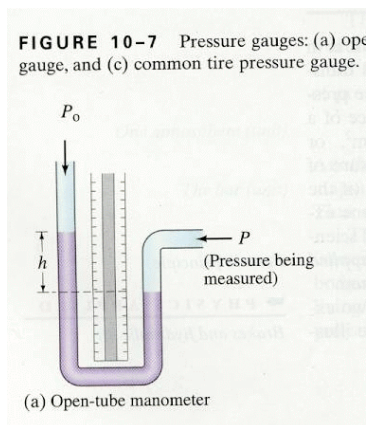
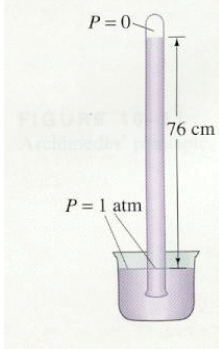


FIGURE 10-8 Diagram of a mercury barometer, when the air pressure is 76 cm-Hg.



open-tube manometer—U-shaped tube partially filled with mercury or water; P being measured is related to the difference in height, h, of the 2 levels

- $P = P_o + Dgh$
- Dgh is the **GAUGE pressure**—the amount by which P exceeds the atmospheric pressure
 - mm Hg or in Hg in this country—often Dgh is NOT calculated and only h is reported [think weather reports]
 - $1 \text{ mm Hg} = 133 \text{ N/m}^2 = 1 \text{ torr}$ since....

$$1 \text{ mm} = 1 \times 10^{-3} \text{ m AND } \rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$$

$$\rho gh = (13.6 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1 \times 10^{-3} \text{ m}) = 133 \text{ N/m}^2$$

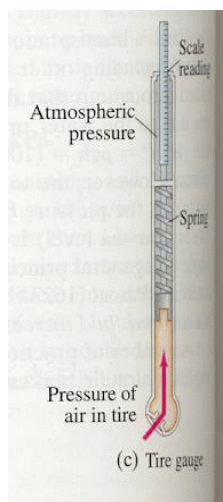
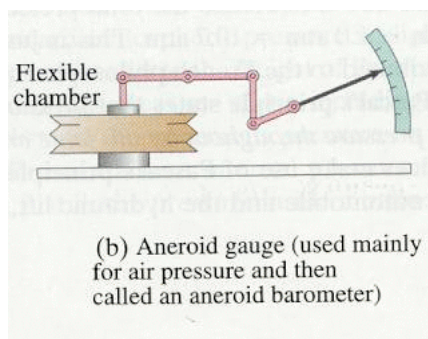
- torr**—1 mm Hg and is in honor of Evangelista Torricelli (1608-1647), the inventor of the barometer

barometer—one end of a tube of mercury is closed, the open end is suspended in a pool of mercury. The atmospheric pressure supports a column of mercury 76 cm (760 mm) tall @ sea level on a clear day.

$$P = Dg h = (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.760 \text{ m}) = 1.013 \times 10^5 \text{ N/m}^2 = 1.0 \text{ atm}$$

TABLE 10-2
Conversion Factors Between Different Units of Pressure

In Terms of $1 \text{ Pa} = 1 \text{ N/m}^2$	Related to 1 atm
$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$ $= 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa}$	$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$1 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \text{ bar}$
$1 \text{ dyne/cm}^2 = 0.1 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1 \text{ lb/in}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 14.7 \text{ lb/in}^2$
$1 \text{ lb/ft}^2 = 47.9 \text{ N/m}^2$	$1 \text{ atm} = 2.12 \times 10^3 \text{ lb/ft}^2$
$1 \text{ cm-Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 76 \text{ cm-Hg}$
$1 \text{ mm-Hg} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ mm-Hg}$
$1 \text{ torr} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ torr}$
$1 \text{ mm-H}_2\text{O} (4^\circ\text{C}) = 9.81 \text{ N/m}^2$	$1 \text{ atm} = 1.03 \times 10^4 \text{ mm-H}_2\text{O} (4^\circ\text{C})$



When other quantities are in SI units, use the Pa as your pressure unit

- **aneroid GAUGE**—pointer linked to the flexible ends of an evacuated thin metal chamber.
 - If electronic, P is applied to a thin metal diaphragm whose resulting distortion is detected electrically
- No matter how good a vacuum pump is, it cannot lift water more than 10 meters.
 - Torricelli was a student of Galileo and explained that a vacuum does NOT suck water up a tube, but rather reduces the P at the top of the tube.

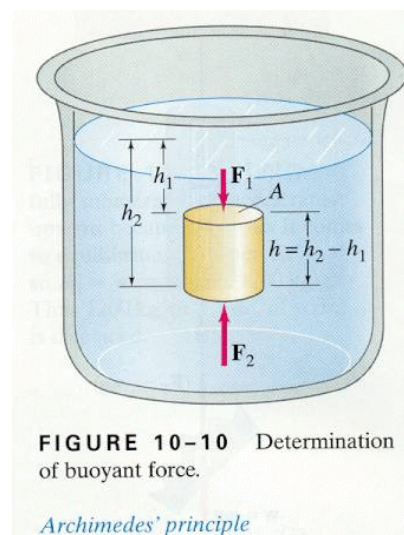
- Atmospheric P *pushes* the water up the tube if its top end is at low P

Example 4

You sit in a meeting where a novice NASA engineer proposes suction cup shoes for Space Shuttle astronauts working on the exterior of the spacecraft. Having just studied this chapter, you gently remind him of the fallacy of this plan. What is it?

BUOYANCY & ARCHIMEDES PRINCIPLE

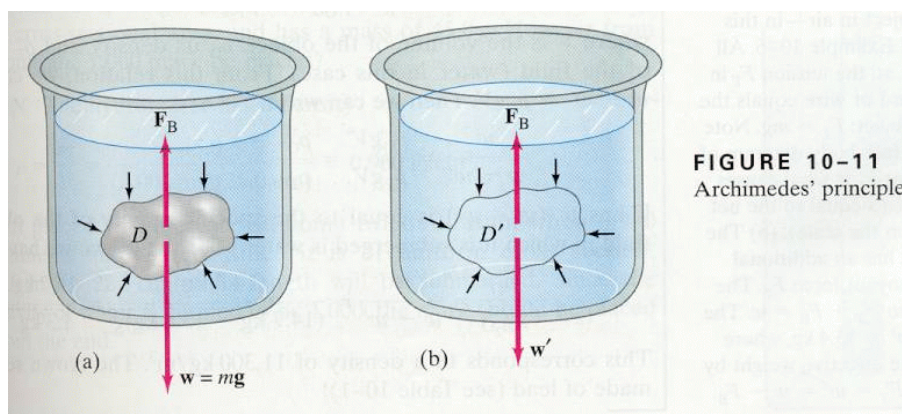
- **buoyancy**—objects in a fluid *appear* to weigh less than when outside the fluid. F_g acts downward while F_b acts upward
 - For fish and divers $F_g > F_b$
 - occurs because P in a fluid increases with depth
 - Consider a cylinder of height, h , whose top and bottom ends have an area A & which is completely submerged in a fluid of density D_f
 - The fluid exerts a pressure $P_1 = D_f g h_1$ at the top surface of the cylinder
 - The force due to this pressure on the top of the cylinder is $F_1 = P_1 A = D_f g h_1 A$ and is directed downward
 - The fluid exerts an upward force on the *bottom* of the cylinder equal to $F_2 = P_2 A = D_f g h_2 A$
 - THE NET FORCE due to fluid pressure is the buoyant force, F_b
 - $F_b = F_2 - F_1$
 - $= D_f g A (h_2 - h_1)$
 - $= D_f g A h$
 - $= D_f g V$ since $V = Ah = \text{volume of the cylinder}$



AND

$$D_f g V = m_f g \quad \hat{=} \quad F_b = \text{weight of fluid displaced by the cylinder}$$

- **Archimedes (287?–212 B.C.) Principle**—the buoyant force on a body *immersed* in a fluid is equal to the weight of the fluid displaced by that object.
- The rock D is acted upon by F_w downward and F_b upward. We want to determine F_b
- Consider a “rock” composed of the fluid D' with the same size and shape as the original rock in (a)



- Since the fluid is at rest, F_b on $D = F_b$ on D' since the surrounding fluid is in EXACTLY the same configuration
- Therefore, $F_b = W'$ so $F_b = \text{weight of the body of the fluid whose volume} = \text{volume of } D$

Example 5

A 70 kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 \text{ cm}^3$. How much force is needed to lift it?

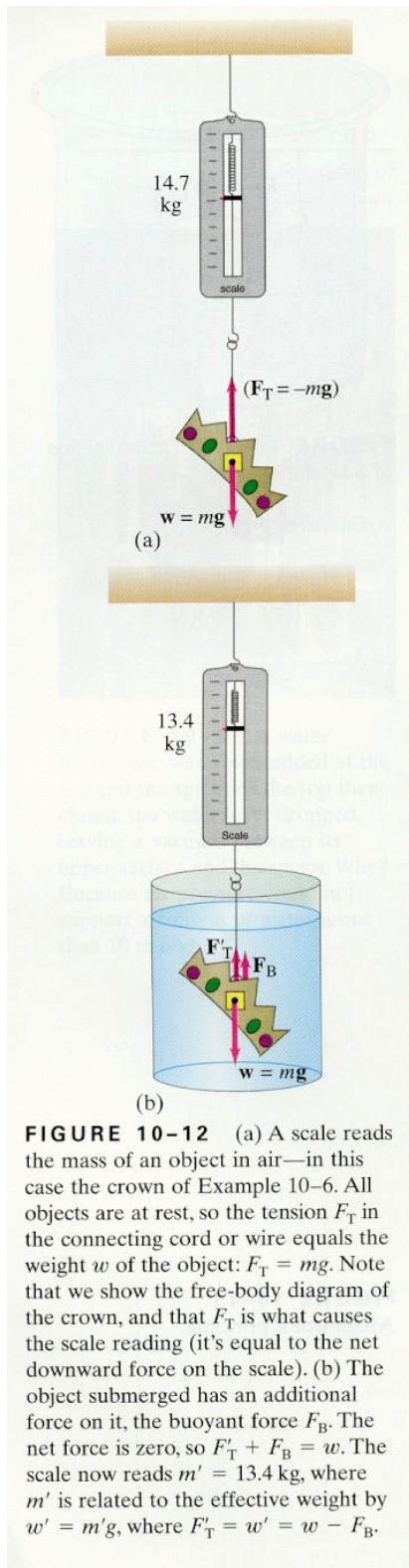


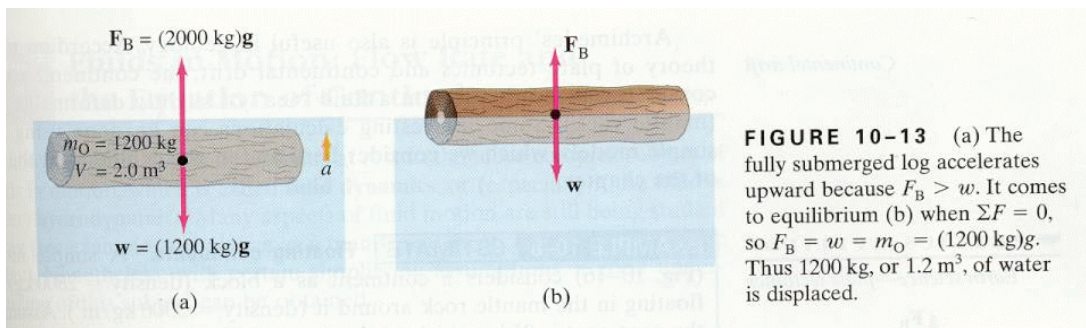
FIGURE 10-12 (a) A scale reads the mass of an object in air—in this case the crown of Example 10-6. All objects are at rest, so the tension F_T in the connecting cord or wire equals the weight w of the object: $F_T = mg$. Note that we show the free-body diagram of the crown, and that F_T is what causes the scale reading (it's equal to the net downward force on the scale). (b) The object submerged has an additional force on it, the buoyant force F_B . The net force is zero, so $F_T + F_B = w$. The scale now reads $m' = 13.4 \text{ kg}$, where m' is related to the effective weight by $w' = m'g$, where $F_T = w' = w - F_B$.

- Archimedes' bath—it is said that Archimedes's figured all of this out as a result of a problem brought to him by the King. The King had a new crown made and wanted Archimedes's to verify that it was made of gold and that the King had not been cheated [penalty of death!]. When getting into his bath tub the solution “came to him”.....
 - $\rho_{\text{gold}} = 19.3$, but the volume of the irregularly shaped crown was very hard to calculate
 - Object weighed in air = w ; an object “weighed” in water has an *apparent weight* = w'
 - $w' = w - F_b = mg - F_b$ & then you can calculate the density and see if it agrees with gold's!

Example 6

When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

- Archimedes's Principle applies to objects that float as well
- An object floats if less dense than surrounding fluid
- At equilibrium, i.e. when floating, $mg = F_b$



- For example: we can determine the mass of a log with a $SG = .60$ & $V = 2.0 \text{ m}^3$
 - $m_o = D_o V = .6 \times 10^3 \text{ kg/m}^3 (2.0 \text{ m}^3) = 1200 \text{ kg}$
 - IF fully submerged, displaces mass of water $m_F = D_F V = 1000 \text{ kg/m}^3 (2.0 \text{ m}^3) = 2,000 \text{ kg}$
 - HENCE $F_b > \text{weight}$ ^ floats. It comes to equilibrium when 1200 kg of water has been displaced, so 1.2 m^3 of its volume will be submerged $(1.2/2.0) = .60$ ^ 60% of the log is submerged
- When an object floats $F_B = W = mg$

$$\rho_F V_{\text{displ}} g = \rho_o V_o g$$

V_o = full volume of the object and V_{displ} = volume of fluid displaced (= volume submerged)

thus...
$$\frac{V_{\text{displ}}}{V_o} = \frac{\rho_o}{\rho_F}$$

- The fraction of the object submerged is given by the ratio of the objects density to that of the fluid.

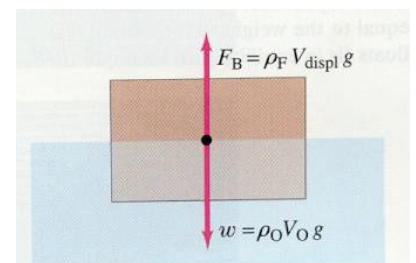
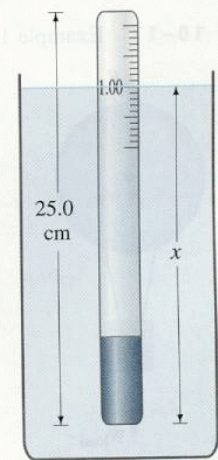


FIGURE 10-14 An object floating in equilibrium: $F_B = w$.

FIGURE 10-15 A hydrometer. Example 10-7.



Example 7

A hydrometer is a simple instrument used to indicate specific gravity of a liquid by measuring how deeply it sinks in the liquid. A particular hydrometer consists of a glass tube, weighted at the bottom, which is 25.0 cm long, 2.00 cm^2 in cross-sectional area, and has a mass of 45.0 g. How far from the end should the 1.000 mark be placed?

- Also useful in geology. According to the theory of plate tectonics and continental drift, the continents can be considered to be floating on a fluid “sea” of slightly deformable rock (mantle rock)

Example 8

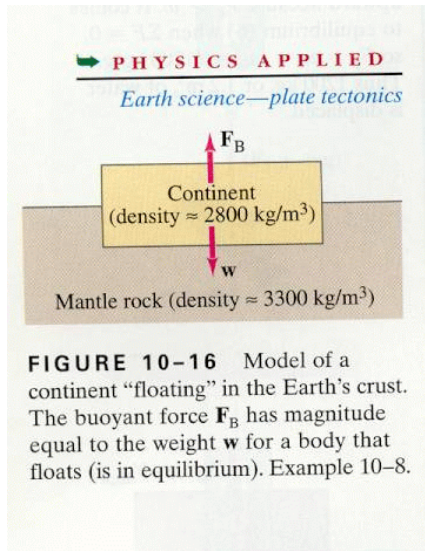


FIGURE 10-16 Model of a continent “floating” in the Earth’s crust. The buoyant force F_B has magnitude equal to the weight w for a body that floats (is in equilibrium). Example 10-8.

A simple model considers a continent as a block (density = 2800 kg/m^3) floating in the mantle rock around it (density = 3300 kg/m^3). Assuming the continent is 35 km thick (the average thickness of the Earth’s crust), estimate the height of the continent above the surrounding rock.

- Air is a fluid too—ordinary objects weigh less in air than in a vacuum.

Example 9

What volume V of helium is needed if a balloon is to lift a load of 800 kg (including the weight of the empty balloon)?

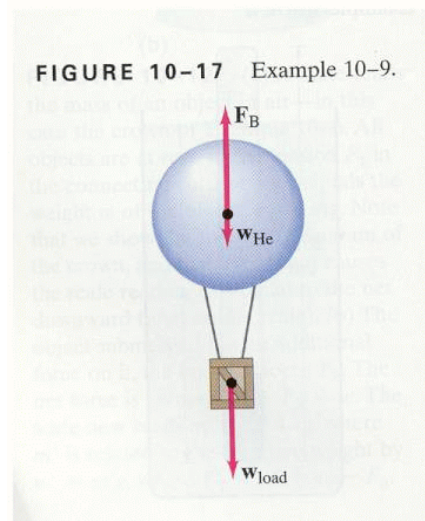


FIGURE 10-17 Example 10-9.

FLUIDS IN MOTION; FLOW RATE & THE EQUATION OF CONTINUITY

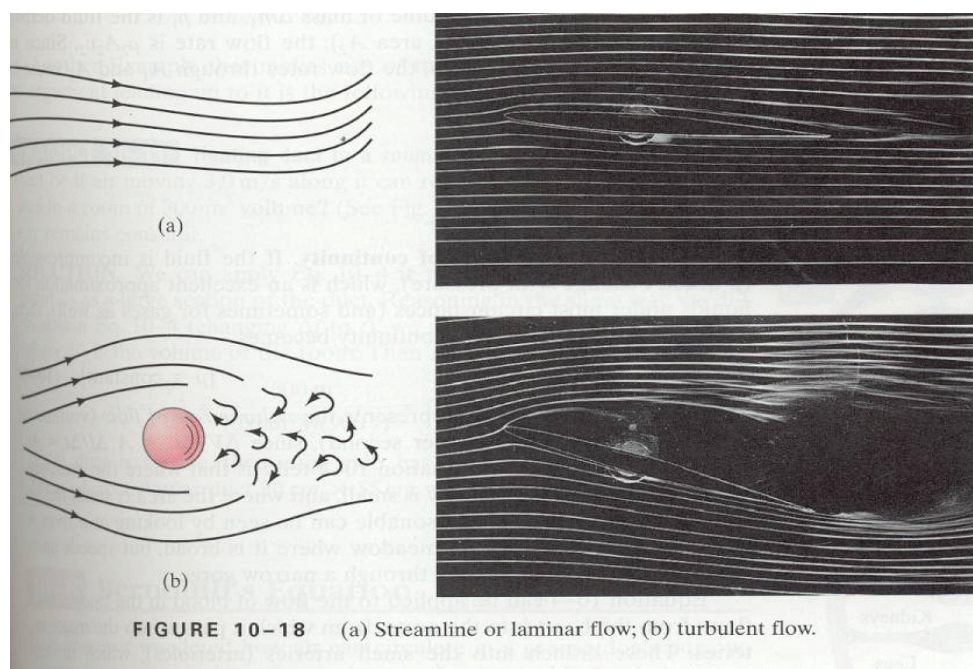


FIGURE 10-18 (a) Streamline or laminar flow; (b) turbulent flow.

- **Fluid dynamics**—fluids in motion, more complex!
- **hydrodynamics**—if the fluid in question is WATER
- **TWO types of fluid flow**
- Streamline or **laminar** flow—flow is smooth, such that neighboring layers of fluid slide by each other smoothly; each particle follows a smooth path without crossing
- **Turbulent flow**—erratic, small whirlpool-like circles called eddy currents or eddies; a function of current speed; Eddies absorb a great deal of E and increase internal friction—viscosity
- Consider a steady laminar flow through an enclosed tube/pipe

How does the speed of the fluid change as the size of the tube changes?

- **mass flow rate**—the mass (m) of fluid that passes a given point per unit time (t). Mass flow rate = $\frac{m}{t}$

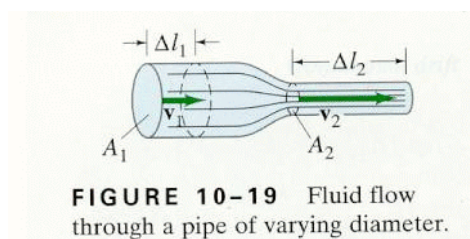


FIGURE 10-19 Fluid flow through a pipe of varying diameter.

The volume of fluid passing point 1 = $A_1 v_1 \Delta t$ where $v_1 = \frac{\Delta \ell_1}{\Delta t}$, the mass flow rate through area 1 is:

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta \ell_1}{\Delta t} = \rho_1 A_1 v_1$$

where $\Delta V_1 = A_1 \Delta \ell_1$

At point 2 the flow rate = $D_2 A_2 v_2$ AND since no fluid flows in or out of the sides, flow rates through A_1 & A_2 MUST be equal, THEREFORE

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \quad \text{THEN} \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

The EQUATION OF CONTINUITY:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

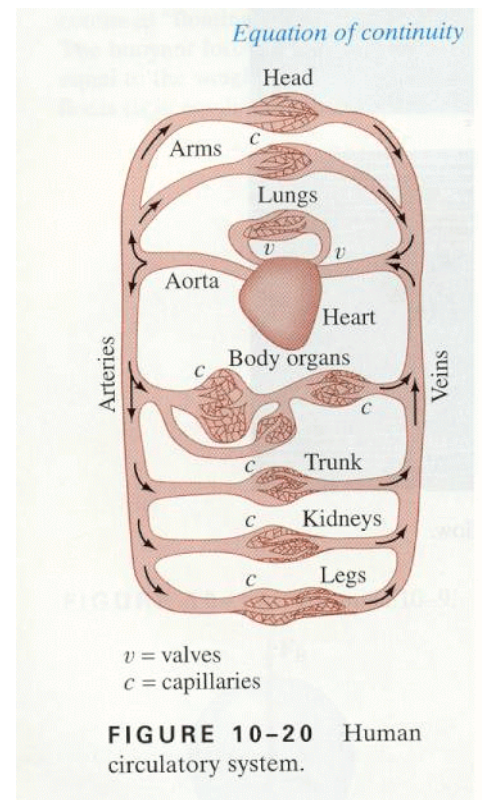
IF the fluid is **IN**compressible, then $D_1 = D_2$ AND $A_1 v_1 = A_2 v_2$
 where $A_1 v_1$ is the **volume** rate of flow in m^3/second

- When the cross-sectional area is large, v is small. Ever squeeze a garden hose? OR put your thumb over the opening to make it smaller and spray the water at an unsuspecting target?
- Blood flow works the same way

Heart ÷ aorta ÷ arteries ÷ arterioles ÷ capillaries ÷ venule ÷ veins ÷ back to heart

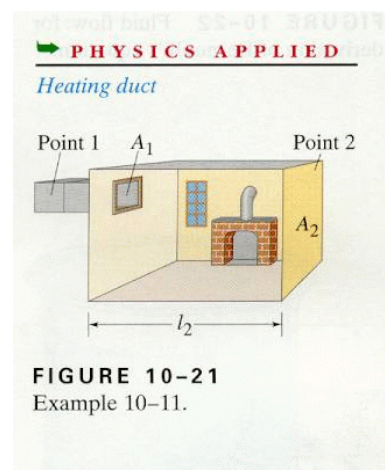
Example 10

The radius of the aorta is about 1.0 cm and the blood passing through it has a speed of about 30 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate how many capillaries there are in the body.



Example 11

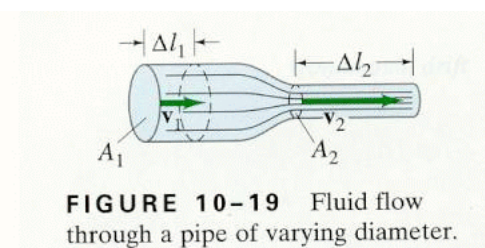
How large must a heating duct be if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of 300 m³ volume? Assume the air's density remains constant.



BERNOULLI'S EQUATION

WHY does smoke go up a chimney? WHY does a rag-top on a convertible bulge up at high speeds?

Daniel Bernoulli (1700-1782)—where the v of a fluid is high, P is low and where v is low, P is high



The pressure at point 2 is lower (since v is greater @ point 2) than at point 1 where v is smaller.

High pressure would slow the fluid down.

To derive Bernoulli's equation, assume flow is steady and laminar, the fluid is incompressible and viscosity is small enough to be ignored.

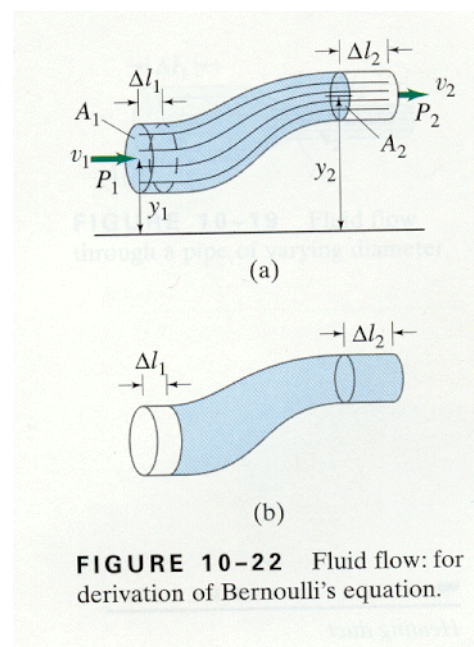
We assume the fluid is flowing in a tube of nonuniform cross-section that varies in height above some reference level

Calculate work of shaded fluid in moving from (a) to (b).

Fluid at point 1 flows a distance $\Delta \ell_1$ and forces the fluid at point 2 to move a distance $\Delta \ell_2$. The fluid to the left of point 1 exerts a pressure P_1 on our section of fluid and does an amount of work

$W_1 = F_1 \Delta \ell_1 = P_1 A_1 \Delta \ell_1$ at point 2, work done is

$W_2 = -P_2 A_2 \Delta \ell_2$ the negative sign is present because of the force exerted on the fluid is *opposite* to the motion. (W is also done by the F_{gravity})



Since the net effect of the process shown is to move a mass m of volume $A_1) R_1 (= A_2) R_2$, since the fluid is incompressible) from point 1 to point 2. The work done by gravity is

$W_3 = -mg(y_2 - y_1)$ y_1 and y_2 are heights of the center of the tube above a reference level. W_3 is negative since the motion is uphill against F_g .

$$W_{\text{net}} = W_1 + W_2 + W_3$$

$$W = P_1 A_1) R_1 - P_2 A_2) R_2 - mgy_2 + mgy_1$$

According to the work energy theorem: $W_{\text{net}} =) KE$, THEREFORE....

$$\frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = P_1 A_1) R_1 - P_2 A_2) R_2 - mgy_2 + mgy_1$$

The mass, m , has a volume $A_1) R_1 = A_2) R_2$. Thus we can substitute $m = D_1 A_1) R_1 = D_2 A_2) R_2$, and also divide through by $A_1) R_1 = A_2) R_2$, to obtain:

$$\frac{1}{2} Dv_2^2 - \frac{1}{2} Dv_1^2 = P_1 - P_2 - Dgy_2 + Dgy_1$$

rearrange to collect all the "1" terms on one side and all the "2" terms on the other and

Bernoulli's Equation:

$$P_1 + \frac{1}{2} Dv_1^2 + Dgy_1 = P_2 + \frac{1}{2} Dv_2^2 + Dgy_2$$

Since points 1 & 2 can be any 2 points along a tube of flow, Bernoulli's equation can be written:

$$P + \frac{1}{2} Dv^2 + Dgy = \text{CONSTANT @ every point in the fluid}$$

Bernoulli's equation is an expression of the law of energy conservation since we derived it from the work-energy principle.

Example 12

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm-diameter pipe on the second floor 5.0 m above?

APPLICATIONS OF B'S PRINCIPLE: From Torricelli to Sailboats, Airfoils & TIA

Classic application:

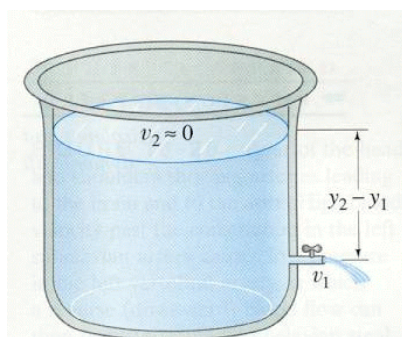


FIGURE 10-23 Torricelli's theorem: $v_1 = \sqrt{2g(y_2 - y_1)}$.

Torricelli's theorem

Point 1 is spigot and point 2 is the top surface. Since both are open to the atmosphere $P_1 = P_2$ so...

$$\frac{1}{2} Dv_1^2 + Dgy_1 = Dgy_2 \quad \text{OR}$$

$$v_1 = \sqrt{2g(y_2 - y_1)}$$

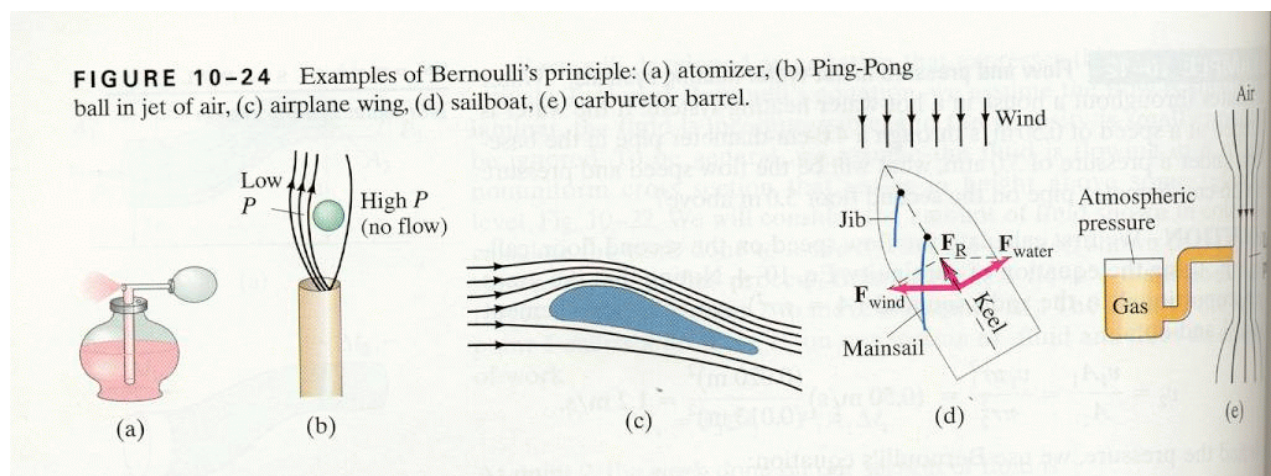
This is called Torricelli's Theorem and was discovered 100 years before Bernoulli's.

The liquid leaves the spigot with a velocity that a free falling object would attain falling from the same height.

When fluid is flowing without an appreciable change in height, $y_1 = y_2$

Then B's equation becomes:

$$P_1 + \frac{1}{2} Dv_1^2 = P_2 + \frac{1}{2} Dv_2^2 \quad \text{As speed increases, P drops}$$



(a) The pressure in the air blown at high velocity across the top of the vertical tube of a perfume atomizer is less than the pressure of the atmosphere, $\hat{~}$ perfume is pushed up due to reduced pressure at the top!

(b) A ping-pong ball floats above a blowing jet of air; if it begins to leave the jet, the increased pressure in the still air outside the jet pushes the ball back in.

(c) Airplane wings and airfoils deflect air so streamline flow is largely maintained. Air speed is greater above the wing $\hat{~}$ P above the wing is less than below and the *net upward* F is called **dynamic lift** [coupled with tilt of wing so that the change in momentum of rebounding air molecules results in additional upward force on the wing]

(d) Sailboat can move against the wind—arrange sails so that the velocity of the air increases in the narrow constriction between the 2 sails. P of the atmosphere behind the mainsail is larger than the reduced pressure in front of it (due to fast moving air in narrow slot between sails) and pushes boat forward. When going against the wind, set mainsail at an angle approximately midway between wind direction and main axis of boat [keel]. The NET force on the sail acts Z to the sail, F_{wind} . The boat moves sideways EXCEPT the keel extends vertically downward beneath the water. Water exerts F_{water} on the keel nearly Z to keel $\hat{}$ resultant F is almost directly forward!

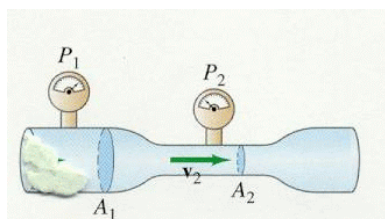
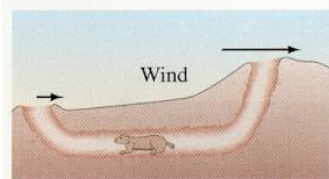


FIGURE 10-26 Venturi meter.

PHYSICS APPLIED

Smoke up a chimney

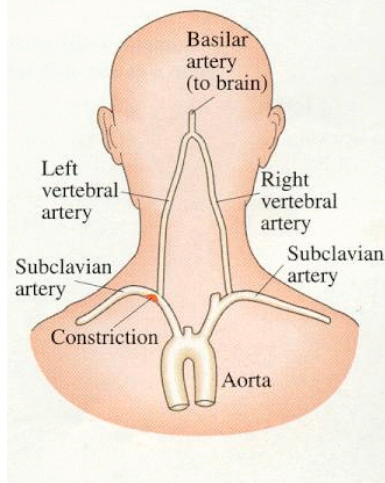
FIGURE 10-27 Bernoulli's principle is responsible for air flow in underground burrows.



PHYSICS APPLIED

Medicine—TIA

FIGURE 10-28 Rear of the head and shoulders showing arteries leading to the brain and to the arms. High blood velocity past the constriction in the left subclavian artery causes low pressure in the left vertebral artery, in which a reverse (downward) blood flow can then result: so-called "subclavian steal syndrome," resulting in a TIA (see text).



(e) **Venturi tube**—barrel of a carburetor flowing air speeds up as it passes this constriction and so the pressure drops. Gasoline under atmospheric P in the carburetor reservoir is forced into the air stream & mixes with air before entering the cylinders. A venturi meter measures the flow speed of fluids

Why does smoke go up a chimney?

Hot air is less dense [since it has expanded] & floats
Wind blows across the chimney top making the P less than the atmospheric pressure in the house, hence air and smoke go up the chimney.

How does air circulate in a rodent's burrow?

2 entrances; The speed of air flow will be slightly different which results in a slight P difference which forces a flow of air through the burrow. A smart rodent builds the entrances at different heights to enhance air flow.

TIA—Transient Ischemic Attack [for the prospective pre-med's out there!]. English translation: Temporary lack of blood supply to the brain caused by the so-called "subclavian steal syndrome". Symptoms: dizziness, double vision, headache and weakness of the limbs.

Blood normally flows up to the brain at the back of the head via 2 vertebral arteries—one on each side of the neck—which meet to form the basilar artery just below the brain.

Vert. arteries issue from the subclavian arteries as shown, before the latter pass to the arms. When an arm is exercised vigorously, blood flow increases to meet the needs of the arm's muscles. IF the subclavian is partially blocked, [too many french fries!] the blood velocity will increase to supply needed blood (smaller A means larger v to equal same rate of flow). Increased blood v past opening of vert. artery because of low P on that side (Venturi effect). Blood should have passed upward into basilar artery and the brain therefore, blood supply to the brain is reduced. The fast moving blood in the subclavian artery "steals" the blood away from the brain. Resulting dizziness or weakness usually causes the person to stop the exertions followed by a return to normal.